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MONTHLY NOTICES

OF THE

ROYAL ASTRONOMICAL SOCIETY.

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Professor Cayley, F.R.S., President, in the Chair.

J. Bridson, Esq., Bolton;
T. W. Bush, Esq., Cromwell Street, Nottingham;
W. H. Finlay, Esq., Royal Observatory, C.G.H.; and
Lieut. G. F. Guyon, St. Leonards-on-Sea,

were balloted for and duly elected Fellows of the Society.

The following Note is inserted by special order of the Council:—

The attention of the Council has been directed to certain remarks made by their late Editor, in paragraph 2, page 583, of the Supplementary Number, Vol. xxxiii. of the *Monthly Notices.* The Council have entered on their minutes a resolution expressing their strong disapprobation of the paragraph referred to.

On the rejection, in the Lunar Theory, of the term of Longitude depending for argument on eight times the mean longitude of Venus minus thirteen times the mean longitude of the Earth, introduced by Professor Hansen; and on the effect of that rejection upon the state of the Lunar Tables, and upon the lunar calculations which serve as basis for Ancient Chronology.

By George Biddell Airy, K.C.B., Astronomer Royal.

In a note, dated 1827, December 16, to a paper published in the *Philosophical Transactions* for 1828, I announced that I had discovered equations of long period in the movements of the

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Earth and *Venus* depending on the argument eight times the mean longitude of *Venus minus* thirteen times the mean longitude of the Earth, of which the period is 239 years; and I gave approximate values of the co-efficients. In a paper, dated 1831, November 8, published in the *Philosophical Transactions* for 1832, I gave the details of the computations. The whole process has been reverified, first by M. le Comte G. de Pontécoulant, secondly by M. Le Verrier, and is found to be correct.

From the first announcement of the discovery of this planetary equation, mathematicians, who were more specially engaged on the lunar theory, considered it probable that there would be found an inequality in the geocentric motion of the Moon depending on the same argument. In the summer of 1829, I visited M. Carlini, in the neighbourhood of Milan, and he immediately made inquiries of me as to the possible existence of such an inequality in the lunar motion. In a paper by M. Poisson, "sur le mouvement de la lune autour de la terre," dated 1833, June 17, and published in the *Mémoires de l'Académie Royale des Sciences de l'Institut de France*, tome xiii. 1835, there was given a calculation of the lunar inequality, which led M. Poisson to the conclusion that the inequality in the Moon's motion is insensible. It was, however, subsequently pointed out by M. Delaunay that Poisson's calculation was based only on the disturbance of the excentricity of the Earth's orbit, whereas the effect of the disturbance of the major axis of the Earth's orbit was more important. In this state the question rested for many years.

In 1847, Professor Hansen announced to the French Academy of Sciences (*Comptes Rendus*, tome xxiv. page 795), that he had discovered two equations of long period in the Moon's motion, depending respectively on the arguments 18 × mean long. *Venus* − 16 × mean long. Earth − Moon's mean anomaly, and 8 × mean long. *Venus* − 13 × mean long. Earth, with periods respectively of 273 and 239 years. At the same time, Professor Hansen communicated to the Royal Astronomical Society a statement of the same general character, of which an extract is published in the *Monthly Notices* of the Society for 1847, May 14. Subsequent investigations (of which, I believe, no detailed accounts are published) induced him to make some alterations in the coefficients first adopted; and, finally, in the Lunar Tables, Professor Hansen adopted the following terms:—

$$+15''\!\cdot\!34 \times \sin\{18.\ \text{mean long. } Venus - 16.\ \text{mean long. Earth} - \text{Moon's mean anomaly} + 30^\circ\ 12'\}$$

$$+21''\!\cdot\!47 \times \sin\{8.\ \text{mean long. } Venus - 13.\ \text{mean long. Earth} + 274^\circ\ 14'\}.$$

The tables, affected with these equations, have been used by myself in the computations of the Eclipses of Thales, Larissa, Agathocles, and Sticklastad, *Memoirs of the Royal Astronomical Society*, vol. xxvi. page 131, &c.

The first of these equations has been recalculated by M. De-

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launay. After a first failure (*Comptes Rendus*, tome xlix. page 293), the examination was repeated, and every step of the investigation is printed with the utmost detail, in the *Additions* to the *Connaissance des Temps*, 1862. M. Delaunay's result agrees substantially with Professor Hansen's. It is unnecessary, therefore, to allude further to this inequality.*

The second equation, also, depending on 8 × mean long. *Venus* − 13 × mean long. Earth, was recalculated by M. Delaunay, but with very different result. His investigations, which are given in detail in the *Additions* to the *Connaissance des Temps* for 1863, lead to the conclusion that the direct effect of *Venus* upon the Moon produces an equation of the form above-mentioned with coefficient $0''\cdot0039$; and that the indirect effect of *Venus*, by altering the Earth's distance from the Sun and its heliocentric longitude (especially the distance), and thus modifying the action of the Sun upon the Moon, produces an equation with coefficient $0''\cdot2723$. I am informed, in private communication from Professor Newcomb, that he has repeated the investigations with nearly the same result. These coefficients, in comparison with that found by Professor Hansen, may be described as insignificant.

Assuming then that the equation

$$+21''\cdot47 \times \sin\{8.\ \text{mean long. } Venus - 13.\ \text{mean long. Earth} + 274^\circ\ 14'\}$$

ought to be withdrawn from Hansen's tables, it becomes matter of interest to ascertain what will be the effect of that withdrawal upon the Lunar Tables and upon the theory embodied in those tables. For this purpose, I must refer to my paper in vol. xxix. of the Society's *Memoirs*, "Corrections of the Elements of the Moon's Orbit, &c.," and specially to the section, beginning on page 9, and headed, "Values of R. and Correction of Moon's Epoch of Longitude and Mean Motion." It must be remembered that the Moon's observed longitudes have been compared with longitudes computed, principally by use of Damoiseau's Tables, with Damoiseau's Mean Longitudes, on Plana's Theory; that the results of the comparisons have been collected in groups of years; and that for each group the corrections of certain elements are computed. The only element which concerns us here is the Moon's Mean Longitude. The correction which must be applied

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* I cannot dismiss this remarkable equation without expressing my opinion that its discovery is one of the greatest steps—perhaps the greatest step—in Gravitational Astronomy. My experience in the calculation of the planetary equation depending on {8. mean long. *Venus* − 13. mean long. Earth} enables me to appreciate the labour of Professor Hansen's calculation. The research by which it was suggested must also have been laborious. It may be interesting to the student of these investigations to remark that the final value of the coefficient depends almost entirely on the inclination of the orbit of *Venus* to the ecliptic. The relative position of the perihelia of *Venus* and the Earth, and the smallness of the excentricities, greatly diminish the terms independent of inclination. I had pointed out this circumstance in the planetary equation to which I have alluded.

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to Damoiseau's Tabular Mean Longitude, in order to produce the Mean Longitude deduced from observation, is called R.

Now this comparison with Plana's Theory and Damoiseau's Mean Longitudes was made before the suggestion by Hansen of the two equations of long period produced by the action of *Venus*. Regarding the results of the means of theoretical terms of short period through a few years, as being essentially the same in the two theories, the mean longitude being always subject to correction, we could then add the numerical values of Hansen's two equations to Plana's Theoretical Longitudes, or we could subtract the values of the two equations from the values of R, in order to ascertain the true correction to Damoiseau's Mean Longitudes, which were used in the comparison of observation with theory. This process was used in the paper to which I have referred, and a series of values of "R − Hansen's Inequalities" is formed on page 9 of that paper, which it is not necessary to reprint here. It is sufficient to state that it was found on page 10, that the errors of the groups might be expressed in the mass by the formula $-2''{\cdot}48 + 2''{\cdot}771 \times$ No. of group, measured from the middle group, and that, on substituting in this formula, the residual errors for the individual groups were not unreasonably large.

But the rejection of one of Hansen's equations invalidates the latter part of this process. It is necessary now to use only the inequality depending on 18 × mean long. *Venus* − 16 × mean long. Earth − Moon's mean anomaly. Mr. Christie has kindly computed for me the value of this equation for the middle of each group; and the determination of the correction to the Tabular Mean Longitude now stands as follows:—

Range of years in each group.	Value of R given by the equations.	Hansen's inequality of long period.	R—Hansen's inequality.
	″	″	″
1750 to 1759	− 3·19	+ 6·55	− 9·74
1755 to 1764	− 1·38	+ 4·91	− 6·29
1760 to 1768	− 0·31	+ 3·38	− 3·69
1765 to 1773	+ 1·43	+ 1·64	− 0·21
1769 to 1778	+ 3·04	+ 0·06	+ 2·98
1774 to 1782	+ 3·40	− 1·53	+ 4·93
1779 to 1787	+ 3·04	− 3·27	+ 6·31
1783 to 1791	+ 3·76	− 4·63	+ 8·39
1788 to 1796	+ 4·48	− 6·28	+ 10·76
1792 to 1801	+ 2·76	− 7·69	+ 10·45
1797 to 1805	+ 0·87	− 9·02	+ 9·89
1802 to 1810	+ 1·13	− 10·38	+ 11·51
1806 to 1815	+ 0·93	− 11·49	+ 12·42
1811 to 1819	− 0·61	− 12·49	+ 11·88
1816 to 1824	− 1·39	− 13·42	+ 12·03
1820 to 1829	− 1·11	− 14·13	+ 13·02
1825 to 1833	− 1·15	− 14·66	+ 13·51
1830 to 1838	− 1·58	− 15·08	+ 13·50
1834 to 1842	− 1·01	− 15·26	+ 14·25
1839 to 1847	+ 0·15	− 15·31	+ 15·46
1843 to 1851	+ 1·87	− 15·21	+ 17·08

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The values in the last column may be best represented in a simple formula (as before), expressing the correction applicable to mean longitude and mean motion, by the expression + 8″·02 + 1″·097 × No. of group, measured from the middle group. The following table exhibits the result of substitution in this formula :—

No. of Group.	Formula	Excess of R—Hansen's Inequality.	No. of Group.	Formula	Excess of R—Hansen's Inequality.	No. of Group.	Formula.	Excess of R—Hansen's Inequality.
	″	″		″	″		″	″
−10	− 2·95	− 6·79	− 3	+ 4·73	+ 3·66	+ 4	+ 12·41	− 0·38
− 9	− 1·85	− 4·44	− 2	+ 5·83	+ 4·93	+ 5	+ 13·51	− 0·49
− 8	− 0·76	− 2·94	− 1	+ 6·92	+ 3·53	+ 6	+ 14·60	− 1·09
− 7	+ 0·34	− 0·55	0	+ 8·02	+ 1·87	+ 7	+ 15·70	− 2·20
− 6	+ 1·44	+ 1·54	+ 1	+ 9·12	+ 2·39	+ 8	+ 16·80	− 2·55
− 5	+ 2·54	+ 2·39	+ 3	+ 10·21	+ 2·21	+ 10	+ 17·89	− 2·43
− 4	+ 3·63	+ 2·68	+ 2	+ 11·31	+ 0·57	+ 11	+ 18·99	− 1·91

It appears, therefore, that after applying the best correction that we are able to give to the assumed mean longitude and mean motion, there still remains a series of unexplained discordances, varying from − 6″·79 to + 4″·93. And each of the discordances here exhibited is based upon about 440 observations; these observations are all reduced by the same sytem of clock-stars and other elements (Bessel's in the *Tabulæ Regiomontanæ*) ; and the tabular places are all computed by the same theory (Plana's, with Damoiseau's epochs).

I conceive it to be totally impossible that a complete and correct theory can leave such large and systematic discordances. And I express my opinion that there is still some serious defect in the Lunar Theory.

Supposing the formula above given to be accepted as a true exhibition of the correction required to Damoiseau's epochs of longitude, it would appear that (remarking that the mean interval of groups is 4·65 years) Damoiseau's annual motion ought to be increased by 0″·236, or his secular motion ought to be increased by 23″·6. In the former investigation (*Memoirs*, vol. xxix., page 10), Damoiseau's secular motion was increased by 59″·6 ; and, as I understand, Hansen's secular motion was adopted from Damoiseau's, thus corrected. It would appear, therefore, that (still on the supposition that the formula above-mentioned is accepted) the secular motion of Hansen's Tables ought to be diminished by 36″. We have now to examine the effects of this diminution on the eclipses of antiquity, which have been computed with Hansen's Tables.

The term which has been removed from the Tables vanishes about the year 1827. The eclipse of Agathocles occurred in the year −309; that of Larissa (as I accept it) in −556; that of Thalès in −584. The intervals backwards, through which the altered

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motion in longitude applies, are, therefore, 2136, 2383, and 2411 years respectively. The alterations of Moon's motion in longitude (adopting the formula of the last paragraphs) are 12′ 49″, 14′ 18″, 14′ 28″. The tabular places of the Moon will be increased by these quantities.

In my paper 'On the Eclipse of Agathocles, the Eclipse at Larissa, and the Eclipse of Thales,' published in the Society's *Memoirs*, vol. xxvi., page 147, I have remarked that the diminution of Secular Acceleration, from the value employed by Professor Hansen to that proposed by Professor Adams, and supported by the investigations of Delaunay and others, would diminish the tabular longitude of the Moon by 160″ for 1000 years. The effect for the intervals of years above given (proportional to the square of the time) would be 12′ 10″, 15′ 8″, and 15′ 30″. This effect, therefore, so nearly balances the other, that the general result may be thus stated. The eclipses, as described in the paper to which I have referred, are accounted for almost equally well, either by Hansen's Tables, including the equation now rejected, and including also Hansen's secular acceleration; or by Hansen's Tables, deprived of that equation, and combined with a secular acceleration diminished to the extent which I have indicated.

My confidence, however, in the certainty of chronological results, derived from lunar calculations, is in some measure shaken by the character of the unexplained inequality in the epochs of Moon's mean longitude, to which I have called attention.

Royal Observatory, Greenwich,
1873, *October* 8.

On the Correction to Hansen's Semi-Diameter of the Moon from Occultations of Stars. By E. Neison, Esq.

In the Appendix to the volume of *Greenwich Observations* for 1864, Sir George Airy has shown, from the computations of Mr. Breen, that the value for the Moon's semi-diameter given by occultations of stars by the Moon, is less than the apparent telescopic semi-diameter by more than two seconds of arc. As the value for the mean telescopic semi-diameter was derived from the Greenwich observations, and was regarded as an accurate determination of the Moon's semi-diameter as viewed with the Greenwich instruments, this difference was looked upon with considerable interest.

Hansen's value for the semi-diameter of the Moon being regarded as expressing very accurately the true semi-diameter of the Moon, it became a matter of interest to determine whether any similar difference occurred with regard to this value and that given by occultations of stars by the Moon. It was, moreover, desirable to extend the period embraced by Mr.